Rapid Spin Deceleration of Magnetized Proto-Neutron Stars via Asymmetric Neutrino Emission

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We calculate the spin deceleration of proto-neutron stars by asymmetric neutrino absorption in the context of a fully relativistic mean field theory. We calculate for the first time the spin deceleration of neutron stars due to asymmetric neutrino absorption in a toroidal magnetic field configurations. We find a surprising effect that the deceleration can much larger for asymmetric neutrino absorption in a toroidal magnetic field than the usually presumed braking due to magnetic dipole radiation. This may explain the observation that magnetars appear to have had a more rapid deceleration in the past.

Magnetic fields play an important role in many astrophysical phenomena. The observed asymmetry in supernova (SN) remnants, pulsar kick velocities [1], and the existence of magnetars [2, 3] all suggest that strong magnetic fields affect the dynamics of core-collapse SN explosions and the velocity [4] that the proto-neutron stars (PNS) receive at birth.

There are now two major supernova explosion scenarios leading to asymmetric morphologies in observed supernova remnants. One is the standing accretion shock instability (SASI)-aided neutrino driven explosion [5, 6]. The other is the magneto-rotational explosion (MRE) [7, 8]. Both mechanisms may also be a source for pulsar kick velocities [9, 10]. The MRE takes place through the extraction of the rotational energy of the PNS via strongly amplified magnetic

fields $\sim 10^{15}$G [39]. Thus, it is expected to leave a magnetar remnant. However, there are many unknown aspects of these scenarios, such as the progenitor and final core rotation and magnetic field profile. Hence, there is not yet a definitive understanding of the observed asymmetry and remnant kick velocities in core collapse supernovae.

Moreover, there is accumulating evidence that magnetars spin down faster than expected [12]. Usually it is supposed that magnetars spin down mainly by dissipating rotational energy into magnetic dipole radiation. However, it is possible that they could have a much more rapid spin deceleration early in their formation. A new source for this deceleration is the main point of the work reported here.

Here, we show that the early spin deceleration of a PNS could result from an asymmetry in the neutrino emission that arises from parity violation in weak interactions [13, 14] and/or an asymmetric distribution of the magnetic field [15] in strongly magnetized PNSs. (However, for an alternative scenario see Ref. [16].) Recent theoretical calculations [17, 18] have suggested that as little $\sim$1% asymmetry in the neutrino emission out of a total neutrino luminosity of $\sim 10^{53}$ ergs is enough to explain the observed pulsar kick velocities. Here we show that such emission could also explain a rapid early spin down rate of magnetars.

Both static and dynamic properties of neutron-star matter have been studied [19–21] at high temperature and density in the context of spherical non-magnetic neutron star models. Such aspects as an exotic phase of strangeness condensation [22–24], nucleon superfluidity [25], rotation-powered thermal evolution [26], a quark-hadron phase transition [27], etc., have been considered. Neutrino propagation has also been studied for PNS matter including hyperons [28]. These theoretical treatments of high-density hadronic matter, however, have not yet considered the effects of strong magnetic fields. Although previous work [17, 18] has studied the effects of magnetic fields on the asymmetry of neutrino emission, the neutrino-nucleon scattering processes were calculated in a non-relativistic framework [17] and only a uniform dipole field configuration was considered.

In our previous papers [29, 30] we discussed the neutrino scattering and absorption cross sections in hot and dense magnetized neutron-star matter (including hyperons) in fully relativistic mean field (RMF) theory [32]. Those papers demonstrated that magnetic fields enhance the scattering cross-sections in the direction parallel to the magnetic field for neutrinos while also reducing the absorption cross-sections in the same direction. When the direction is anti-parallel, the opposite

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[39] A few processes can be candidates for the amplification mechanisms such as winding effect or the magneto-rotational instability [11].

occurs.

It was shown in Ref. [30] that for interior magnetic field strengths near the equipartition limit $B \sim 2 \times 10^{17}$G, the enhancement of the scattering cross-sections is $\sim 1\%$ at a baryon density of $\rho_B = 3\rho_0$, while the reduction in the absorption cross section is $\sim 2\%$. This enhancement and reduction were shown to increase the neutrino momentum flux emitted along the direction of the magnetic field and to decrease the emitted momentum flux emitted antiparallel to the magnetic field. This asymmetry was then applied to a calculation of pulsar-kick velocities in the context of a one-dimensional Boltzmann equation including only the dominant effect of neutrino absorption. PNS kick velocities of ~ 610 km·s^{-1} were estimated for a neutron star of baryon mass $M_{NS} = 1.68 M_\odot$, a uniform dipole magnetic field of $B = 2 \times 10^{17}$G, an isothermal temperature of $T = 20$MeV, and a total emitted neutrino energy of $E_T \approx 3 \times 10^{53}$erg.

However, magneto-hydrodynamic (MHD) PNS simulations (e.g. [31]) have shown that the magnetic field inside a neutron star may have a toroidal configuration . Here, we apply the above results on asymmetric neutrino absorption to this case. One can easily suppose that neutrinos are preferentially emitted along a direction opposite to that of the rotation. This would enhance the spin down rate of a PNS. In this letter, we present for the first time a study of the effect of asymmetric neutrino absorption on the spin deceleration of PNSs.

Even a strong magnetic field has less mass-energy than the baryonic chemical potential in degenerate neutron-star matter, i.e. $\sqrt{eB} \ll \mu_b$, where μ_b is the baryon chemical potential. Hence, we can treat the magnetic field as a perturbation. We then ignore the contribution from convection currents and consider only the spin-interaction. We also assume that $|\mu_b B| \ll E_b^*(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + M_b^{*2}}$, and treat the single particle energies and the wave function in a perturbative way.

In this framework we then obtain the wave function in a magnetic field by solving the Dirac equation:

$$[\gamma_\mu p^\mu - M_b^* - U_0(b)\gamma_0 - \mu_b B \sigma_z] u_b(p, s) = 0, \tag{1}$$

where $M_b^* = M_b - U_s(b)$, and $U_s(b)$ and $U_0(b)$ are the scalar mean-field and the time-component of the vector mean-field for the baryons b. These scalar and vector fields are calculated in the context of RMF theory.

In Refs. [29, 30] we calculated the absorption cross-section σ_A in PNS matter for an interior magnetic field strength of $B = 2 \times 10^{17}$G and a temperature of $T = 20$ MeV. In those results we demonstrated that the absorption cross-sections are suppressed in the direction parallel to the magnetic field $\boldsymbol{B}$ by about $2 - 4\%$ in the density region of $\rho_B = (1 - 3)\rho_0$. The opposite

effect occurs in the anti-parallel direction. The net effect of these changes in the absorption cross sections leads to an increase in the emitted neutrino momentum flux along the direction of the magnetic field and a decrease of the momentum flux emitted in the antiparallel direction.

However, it is quite likely [31] that the magnetic field exhibits a toroidal configuration within the PNS. Hence, we now consider the implications of a toroidal field configuration on neutrino transport in a strongly magnetized PNS. For this purpose we solve for the neutrino phase-space distribution function $f_\nu(\boldsymbol{r}, \boldsymbol{k})$ using a Boltzmann equation as described below and in Ref. [30].

We assume that the system is static and nearly in local thermodynamic equilibrium. Under this assumption the phase-space distribution function satisfies $\partial f_\nu/\partial t = 0$ and can be expanded as $f_\nu(\boldsymbol{r}, \boldsymbol{k}) = f_0(\boldsymbol{r}, \boldsymbol{k}) + \Delta f(\boldsymbol{r}, \boldsymbol{k})$, where the first term is the local equilibrium part, and the second term is its deviation.

Furthermore, we assume that only the dominant effect of absorption contributes to the neutrino transport, and that the neutrinos travel along a straight line. The Boltzmann equation for f_ν then becomes

$$
\begin{aligned}
\hat{k} \cdot \frac{\partial}{\partial \boldsymbol{r}} f_\nu(\boldsymbol{r}, \boldsymbol{k}) &= \hat{k} \cdot \frac{\partial \varepsilon_\nu(\boldsymbol{r})}{\partial \boldsymbol{r}} \frac{\partial f_0}{\partial \varepsilon_\nu} + \hat{k} \cdot \frac{\partial \Delta f}{\partial \boldsymbol{r}} \\
&\approx -\frac{\sigma_A(\boldsymbol{r}, \boldsymbol{k})}{V} \Delta f(\boldsymbol{r}, \boldsymbol{k}), \quad (2)
\end{aligned}
$$

where $\varepsilon_\nu(\boldsymbol{r})$ is the neutrino chemical potential at coordinate $\boldsymbol{r}$.

Here, we define the variables $x_L \equiv (\boldsymbol{r} \cdot \boldsymbol{k})/|\boldsymbol{k}|$ and $\boldsymbol{R}_T \equiv \boldsymbol{r} - (\boldsymbol{r} \cdot \boldsymbol{k})\boldsymbol{k}/k^2$. We can then solve Eq. (2) analytically
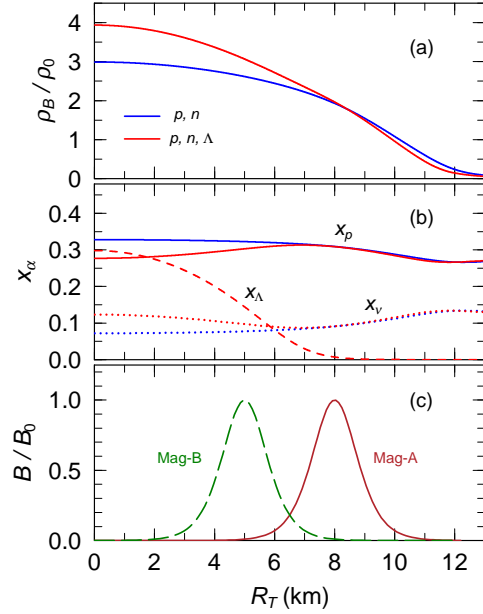

FIG. 1: (Color online) The upper panel (a) shows the baryon density distribution for a PNS with $T = 20$ MeV and $Y_L = 0.4$. The solid and long-dashed lines show results with and without Λs, respectively. The middle panel (b) shows the number fractions for protons, lambdas and neutrinos in a PNS. The solid and dashed lines show the proton fraction in systems with and without Λs. The dot-dashed line indicates the lambda fraction. The dashed and dotted lines denote the neutrino fraction in systems with and without Λs, respectively. The lower panel (c) shows the field strength distribution at $z = 0$ for the toroidal magnetic fields considered here. The solid and dashed lines represent those for $r_0 = 8$km (Mag-A) and 5km (Mag-B), respectively.

$$
\Delta f(x_L, R_T, \boldsymbol{k}) = \int_0^{x_L} dy \left[-\frac{\partial \varepsilon_\nu}{\partial x_L} \frac{\partial f_0}{\partial \varepsilon_\nu} \right] \exp\left[-\int_y^{x_L} dz \frac{\sigma_A(z, R_T, \boldsymbol{k})}{V} \right], \quad (3)
$$

where the center of the neutron-star is at $\boldsymbol{r} = (0,0,0)$, and all of the integrations are performed along a straight line.

In this work we utilize an equation of state (EOS) at a fixed temperature and lepton fraction by using the parameter set PM1-L1 [33] for the RMF as in previous work [29, 30].

We show the baryon density in Fig. 1a and the particle fractions in Fig. 1b as a function of the neutron-star radius. For this figure we assume a neutron star baryonic mass of $M_{NS} = 1.68 M_\odot$, a temperature of $T = 20$MeV, and a lepton fraction of $Y_L = 0.4$. The moment of inertia of the neutron star becomes $I_{NS} = 1.54 \times 10^{45}$ g·cm^2 or 1.36×10^{45} g·cm^2 with or without lambdas, respectively.

One can see in Figs. 1a and 1b the appearance of lambdas for a baryon density greater than about twice the saturation density of nuclear matter, i.e. $\rho_B \gtrsim 2\rho_0$, where $\rho_0 \approx 2.7 \times 10^{14}$ g cm^{-3}. This softens the EOS and leads to an increase in the baryon density and neutrino fraction for $r \lesssim 8$km relative to hadronic matter without lambdas.

The ratio of the total rate of angular momentum loss to the total power radiated by neutrinos at a given spherical surface S_N is

$$\left(\frac{cdL_z/dt}{dE_T/dt} \right) = \frac{\int_{S_N} d\Omega_r \int \frac{d^3k}{(2\pi)^3} \Delta f(\boldsymbol{r},\boldsymbol{k})(\boldsymbol{r} \times \boldsymbol{k}) \cdot \boldsymbol{n}}{\int_{S_N} d\Omega_r \int \frac{d^3k}{(2\pi)^3} \Delta f(\boldsymbol{r},\boldsymbol{k}) \boldsymbol{k} \cdot \boldsymbol{n}}, \tag{4}$$

where $\boldsymbol{n}$ is the unit vector normal to S_N. For illustration we will consider surfaces for which $\rho_B = \rho_0$, and $\rho_B = \rho_0/10$.

In this model the neutrino current is assumed to be constant. Hence, we can use the chain rule to write $dA/dt = cdA/dx_L$ ($A = L_z, E_T$) on the boundary surface, S_N. We can then obtain the angular acceleration from the neutrino luminosity, $\mathcal{L}_\nu = (dE_T/dt)$,

$$\dot{\omega} = -\frac{1}{cI_{NS}} \left(\frac{cdL_z/dt}{dE_T/dt} \right) \mathcal{L}_\nu . \tag{5}$$

For a NS spin period P, the angular velocity is $\omega = 2\pi/P$, and the angular acceleration is $\dot{\omega} = -2\pi \dot{P}/P^2$. Thus, we obtain

$$\frac{\dot{P}}{P} = \frac{P}{2\pi c I_{NS}} \left(\frac{cdL_z/dt}{dE_T/dt} \right) \mathcal{L}_\nu . \tag{6}$$

We adopt the following parameterization of the toroidal magnetic field configuration,

$$\vec{B} = B_0 G_L(z) G_T(r_T) \hat{e}_\phi, \tag{7}$$

where $\hat{e}_\phi = (-\sin\phi, \cos\phi, 0)$ in terms of the azimuthal angle ϕ, and

$$G_L(z) = \frac{4e^{z/a_0}}{\left[1 + e^{z/a_0}\right]^2}, \qquad G_T(r_T) = \frac{4e^{(r_T-r_0)/a_0}}{\left[1 + e^{(r_T-r_0)/a_0}\right]^2}. \tag{8}$$

For our purposes we assume that the toroidal magnetic field is aligned along the direction of the spin rotation. Toroidal fields could however be oppositely oriented and can even invert with time. The generalization to other orientations would imply that neutrino emission could also accelerate the spin of the star. Another complication is the realistic case of a double torus configuration for the magnetic fields. Assuming that the directions of the toroidal magnetic field and the spin rotation are the same in the northern hemisphere, our effects would brake and decelerate the rotation. In the southern hemisphere, however, the direction could be anti-parallel and the asymmetry in neutrino absorption may even accelerate the rotation. This might lead to a complicated twisting mode that could induce p-mode oscillations.

In Fig. 1c we show the magnetic field strength $|\boldsymbol{B}/B_0|$ for different field configurations, with $a_0 = 0.5$ km and $r_0 = 8.0$ km (Mag-A) or $r_0 = 5.0$ km (Mag-B). We assume typical values for the neutrino luminosity $\mathcal{L}_\nu \approx 3 \times 10^{52}$ erg·s^{-1} [18] and a magnetar spin period of $P = 10$ s [34, 35]. We use $B_0 = 2 \times 10^{17}$G and summarize our results in Table I.

	Comp.	$\frac{dcL_z/dt}{dE_T/dt}$	$\dot{P}/P$ (s^{-1})		
			$\rho_B = \rho_0$	$\rho_B = \rho_0/10$	MDR
Mag-A	p, n	66.9	7.82×10^{-2}	1.03×10^{-2}	7.76×10^{-12}
	p, n, Λ	109	1.13×10^{-1}	1.11×10^{-2}	9.86×10^{-12}
Mag-B	p, n	9.64	1.13×10^{-3}	4.50×10^{-5}	7.76×10^{-12}
	p, n, Λ	7.81	8.07×10^{-4}	2.29×10^{-5}	9.86×10^{-12}

TABLE I: The 1st column gives the model for the toroidal magnetic field configuration (see text). The 2nd column shows the presumed composition of nuclear matter, i.e. "p, n" for nucleonic and "p, n, Λ" for hyperonic matter. The 3rd column denotes results from Eq. (4), the 4th and 5th columns are results obtained using Eq. (6) at the indicated baryon density. The 6th column shows the usually presumed spin-down rate from magnetic dipole radiation [Eq. (9)].

To illustrate the density dependence of this effect we include the results obtained for S_N located at $\rho_B = \rho_0/10$ in the fifth column.

For reference the last column of Table I shows $\dot{P}/P$ calculated with the usual magnetic dipole radiation (MDR) formula [36],

$$PP = B^2 \left(\frac{3I_{NS}c^3}{8\pi^2 R^6}\right)^{-1} = B^2 \left(\frac{3M_{NS}^3 c^3}{125\pi^2 I_{NS}^2}\right)^{-1} . \tag{9}$$

Here R is the NS radius whose value is related to the moment of inertia, $I_{NS} = 2M_{NS}R^2/5$. Since MDR derives from the external magnetic field we utilize a typical exterior magnetar poloidal magnetic field of $B = 10^{15}$G in Eq. (9). From this we see that the spin deceleration

from asymmetric neutrino emission can be much larger than that for MDR when the neutrino luminosity is high.

In this calculation we have ignored the neutrino scattering and production processes. The neutrino scattering process enhances the asymmetry of the emission, although its contribution to the mean-free path is much smaller than that from absorption in the density region of interest, $\rho_0 \lesssim \rho_B \lesssim 3\rho_0$ [30].

Neutrino production in a magnetic field is known to cause asymmetry in the neutrino emission [37, 38]. The cross-section for the neutrino production reaction, $e^- + p \rightarrow n, \Lambda + \nu_e$, is qualitatively the same as that for the absorption reaction, $\nu_e + n, \Lambda \rightarrow p + e^-$. The only difference is the small contribution from the magnetic part of the initial and final electron states. Hence, this production process would tend to enhance the asymmetry and to also contribute to the spin deceleration.

Even so, this estimation of $\dot{P}/P$ still has some uncertainties due to unknown variables such as the interior strength of the magnetic field, the spin period of the NS core, etc. Nevertheless, since our value of $\dot{P}/P$ is at least 10^5 times larger than that for MDR, the asymmetric emission could be significant under many possible early conditions. Moreover, other processes such as the neutrino scattering and production tend to increase the spin deceleration. Thus, we can conclude that asymmetric neutrino emission from PNSs may play a very important role in the spin deceleration of a magnetic PNS.

In summary, we have estimated the spin down of a PNS with a toroidal magnetic field configuration by considering the asymmetric neutrino absorption cross-sections calculated in the context of RMF theory. We then solved the Boltzmann equation using a one-dimensional attenuation method, assuming that the neutrinos propagate along an approximately straight line, and that the system is in quasi-equilibrium. We only included neutrino absorption which dominates [30] over scattering in producing asymmetric momentum transfer to the PNS. We found that asymmetric neutrino emission can have a very large effect on the early spin deceleration of a PNS. Indeed, this effect can be much larger than the braking from a magnetic dipole field configuration.

From Eq. (6) we can see that $\dot{P}/P$ from asymmetric neutrino emission is proportional to P, while $\dot{P}/P \propto P^{-2}$ for MDR. This fact implies that asymmetric neutrino emission could play a significant role well into the later stages of core-collapse supernovae.

Finally, we caution that definitive conclusions should involve a fully dynamical MHD simulation of the evolution of a PNS with asymmetric neutrino scattering and production as well as ab-

sorption in a strong magnetic field. Nevertheless, the results presented here make the case for a significant influence of asymmetric neutrino absorption on the process of magnetar formation.

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